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DIFFERENCE SCHEME AND NUMERICAL SIMULATION BASED ON MIXED FINITE ELEMENT METHOD FOR NATURAL CONVECTION PROBLEM *

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Abstract: *The non-stationary natural convection problem is studied. A lowest order finite difference scheme based on mixed finite element method for non-stationary natural convection problem, by the spatial variations discreted with finite element method and time with finite difference scheme was derived, where the numerical solution of velocity, pressure, and temperature can be found together, and a numerical example to simulate the close square cavity is given, which is of practical importance.*

Key words: nutural convection equation; mixed element method; finite difference scheme

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Introduction

Let $\Omega \subset R^2$ be a bounded domain. We consider the following non-stationary natural convection problem:

Problem (I) Find $\boldsymbol{u} = (u_1, u_2)$, p, and T such that, for any $t_1 > 0$,

$$\begin{cases} \boldsymbol{u}_t - \mu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla p = \lambda jT & ((x,y,t)\in\Omega\times(0,t_1)), \\ \mathrm{div}\boldsymbol{u} = 0 & ((x,y,t)\in\Omega\times(0,t_1)), \\ T_t - \Delta T + \lambda\boldsymbol{u}\cdot\nabla T = 0 & ((x,y,t)\in\Omega\times(0,t_1)), \\ \boldsymbol{u} = 0,\ T = 0 & ((x,y,t)\in\partial\Omega\times(0,t_1)), \\ \boldsymbol{u}(x,y,0) = 0,\quad T(x,y,0) = f(x,y) & ((x,y)\in\Omega), \end{cases}$$

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where $\boldsymbol{u}$ is the fluid velocity vector field, p the pressure field, T the temperature field, $\mu > 0$ the coefficient of the kinematic viscosity, $\lambda > 0$ the Groshoff number, $\boldsymbol{j} = (0,1)$ the two-dimensional vector, $f(x,y)$ the given initial scale function.

The natural convection Problem (Ⅰ) is an important system of equation in atmospheric dynamics and a compelling dissipative nonlinear system of equation. Since this system of equation contains not only velocity vector field and the pressure field but also contains temperature field, finding numerical solution is not easy. Even though there are some finite difference schemes treating Problem (Ⅰ) (see Refs. [1 ~ 4] and the references therein), those schemes almost consider the temperature T as a given constant, i.e., regard Problem (Ⅰ) as the Navier-Stokes problem, and introduce vortex function so that the pressure must introduce extra boundary condition. Even so those numerical solutions are no ideal, especially the computing of numerical solution of pressure is usually very difficult. In this paper, a lowest order finite difference scheme based on mixed finite element (MFE) method is derived, where the numerical solution of velocity, pressure, and temperature can be found together, and a result of numerical simulation of the close square cavity is practised, which is of practical importance and shows that our scheme is very effective and trustworthy.

The article is organized as follows: In Section 1, the existence of generalized solution, and the existence and convergency of stable MFE solution for non-stationary natural convection problem are first recalled. And then, in Section 2, a lowest order finite difference scheme based on MFE method are derived. And finally, a numerical example to simulate the close square cavity whose viscosity coefficient μ is very small is given.

1 Recall the Existence of Generalized Solution, and the Existence and Convergency of Stable MFE Solution

The Sobolev spaces and their norms used in context are standard (cf. Ref. [5]).

The generalized solution for non-stationary natural convection Problem (Ⅰ) can be written as

Problem (Ⅰ)* Find $(\boldsymbol{u}, p, T) \in [L^2(0,t_1;X) \cap H^1(0,t_1;V)] \times L^2(0,t_1;M) \times H^1(0,t_1;W)$ such that

$$\begin{cases} (\boldsymbol{u}_t, \boldsymbol{v}) + \mu a(\boldsymbol{u}, \boldsymbol{v}) + a_1(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{v}) - b(p, \boldsymbol{v}) = \lambda \boldsymbol{j}(T, \boldsymbol{v}) & (\forall \boldsymbol{v} \in X), \\ b(\varphi, \boldsymbol{u}) = 0 & (\forall \varphi \in W), \\ (T_t, \psi) + d(T, \psi) + \lambda a_1(\boldsymbol{u}, T, \psi) = 0 & (\forall \psi \in W), \\ \boldsymbol{u}(x,y,0) = 0, \quad T(x,y,0) = f(x,y) & ((x,y) \in \Omega), \end{cases} \tag{1}$$

where

$$X = H_0^1(\Omega)^2, \quad W = H_0^1(\Omega), \quad V = \{\boldsymbol{v} \in X;\ \mathrm{div}\boldsymbol{v} = 0\},$$

$$M = L_0^2(\Omega) = \left\{\varphi \in L^2(\Omega); \int_\Omega \varphi \mathrm{d}x\mathrm{d}y = 0\right\},$$

$$a(\boldsymbol{u}, \boldsymbol{v}) = (\nabla \boldsymbol{u}, \nabla \boldsymbol{v}) = \int_\Omega \left(\frac{\partial u_1}{\partial x}\frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y}\frac{\partial v_1}{\partial y} + \frac{\partial u_2}{\partial x}\frac{\partial v_2}{\partial x} + \frac{\partial u_2}{\partial y}\frac{\partial v_2}{\partial y}\right)\mathrm{d}x\mathrm{d}y,$$

$$a_1(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}) = \int_\Omega \left(u_1 \frac{\partial v_1}{\partial x} w_1 + u_1 \frac{\partial v_2}{\partial x} w_2 + u_2 \frac{\partial v_1}{\partial y} w_1 + u_2 \frac{\partial v_2}{\partial y} w_2\right)\mathrm{d}x\mathrm{d}y,$$

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$$a_1(\boldsymbol{u}, T, \psi) = \int_\Omega \left(u_1 \frac{\partial T}{\partial x}\psi + u_2 \frac{\partial T}{\partial y}\psi \right) \mathrm{d}x\mathrm{d}y,$$

$$b(\varphi, \boldsymbol{v}) = \int_\Omega \varphi \mathrm{div}\boldsymbol{v} \mathrm{d}x\mathrm{d}y,$$

$$d(T, \psi) = (\nabla T, \nabla \psi) = \int_\Omega \left(\frac{\partial T}{\partial x}\frac{\partial \psi}{\partial x} + \frac{\partial T}{\partial y}\frac{\partial \psi}{\partial y} \right) \mathrm{d}x\mathrm{d}y.$$

It has been proved in Ref.[6] that Problem (I)* has a unique solution.

In the following, the spatial variations are discreted with MFE and time with finite difference. Let $\Im_h$ be a quasi-uniform regular triangulation of $\bar{\Omega}$ (see Refs.[8 ~ 9]), and MFE spaces be taken as

$$X_h = \{(v_{1h}, v_{2h}) \in L^2(\Omega)^2;\ v_{jh|K} \in P_1(K) \cap H^1(K)\ \forall K \in \Im_h, v_{jh}n_l^1|_e = v_{jh}n_l^2|_e,\ e = \partial K_1 \cap \partial K_2 \subset \Omega,\ v_{jh}n_j|_e = 0, e \subset \partial K \cap \partial\Omega, i,j = 1,2\}, \tag{2}$$

$$M_h = \{\varphi \in M;\ \varphi|_K \in P_0(K), \forall K \in \Im_h\}, \tag{3}$$

$$W_h = \{\phi \in W;\ \phi|_K \in P_1(K), \forall K \in \Im_h\}, \tag{4}$$

where $P_m(K)$ denotes the space of polynomials of degrees less or equal to m, $\boldsymbol{n}_l = (n_l^1, n_l^2)$ denotes the unite outwards normal on $K_l \in \Im_h$.

It is easy to prove that $X_h \subset X$ (see Ref.[6]). Using the same technique as Ref.[7] one can prove that X_h and M_h satisfy inf-sup condition:

$$\sup_{\boldsymbol{v}_h \in X_h} \frac{b(\varphi_h, \boldsymbol{v}_h)}{\|\nabla \boldsymbol{v}_h\|_0} \geqslant \beta \|\varphi_h\|_0 \qquad (\forall \varphi_h \in M_h), \tag{5}$$

where $\beta > 0$ is a constant independent of h and k. And let L be a positive integer, $k = t_1/L$ be the step width of time, $t^{(n)} = nk$, $0 \leqslant n \leqslant L$; $(\boldsymbol{u}_h^n, p_h^n, T_h^n) \in X_h \times M_h \times W_h$ the MFE approximation corresponding to $(\boldsymbol{u}(t^{(n)}), p(t^{(n)}), T(t^{(n)})) \equiv (\boldsymbol{u}^n, p^n, T^n)$. Then, the fully discrete MFE solution for Problem (I)* can be written as

Problem (I_h^n) Find $(\boldsymbol{u}_h^n, p_h^n, T_h^n) \in X_h \times M_h \times W_h$, $1 \leqslant n \leqslant L$, such that

$$\begin{cases} (\boldsymbol{u}_h^n, \boldsymbol{v}_h) + k\mu a(p_h^n, \boldsymbol{v}_h) - kb(p_h^n, \boldsymbol{v}_h) = k\lambda(jT_h^n, \boldsymbol{v}_h) + (\boldsymbol{u}_h^{n-1}, \boldsymbol{v}_h) - \\ \qquad ka_1(\boldsymbol{u}_h^{n-1}, \boldsymbol{u}_h^{n-1}, \boldsymbol{v}_h) \qquad (\forall \boldsymbol{v}_h \in X_h), \\ b(\varphi_h, \boldsymbol{u}_h^n) = 0 \qquad (\forall \varphi_h \in M_h), \\ (T_h^n, \psi_h) + kd(T_h^n, \psi_h) = (T_h^{n-1}, \psi_h) - \lambda k a_1(\boldsymbol{u}_h^{n-1}, T_h^{n-1}, \psi_h) \qquad (\forall \psi_h \in W_h), \\ T_h^0 = P_h f, \quad \boldsymbol{u}_h^0 = 0, \end{cases} \tag{6}$$

where $P_h f$ is the L^2-projection of f onto W_h.

It is proved in Ref.[4] that Problem (I_h^n) has a unique stable solution and satisfies

$$k^{1/2}\left(\sum_{i=1}^n \|\nabla(T^i - T_h^i)\|_0 + \sum_{i=1}^n \|\nabla(\boldsymbol{u}^i - \boldsymbol{u}_h^i)\|_0 + \|p^n - p_h^n\|_0 \right) + \|\boldsymbol{u}^n - \boldsymbol{u}_h^n\|_0 + \|T^n - T_h^n\|_0 \leqslant C(h + k), \tag{7}$$

where C is a constant independent of h and k.

2 A Lowest Order Difference Scheme Based on MFE Method

The purpose of this section is to derive a lowest order finite difference scheme based on MFE method for Problem (I_h^n). To this end, we cut the domain $\bar{\Omega}$ into a quasi-uniform regular and right triangle set $\Im_h$, and the vertices of each right triangle K is numbered as 1, 2, 3 according to

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counter hands of a clock, the right vertex of K is numbered as $\underline{2}$, see Fig.1.

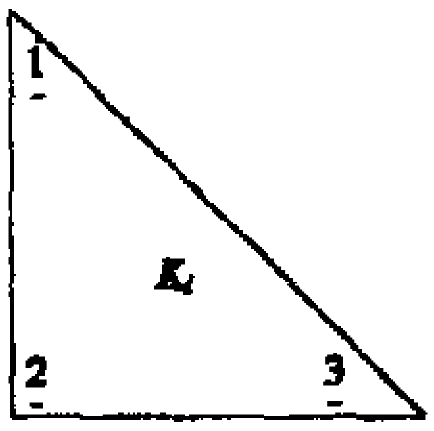

Fig.1 Schematic representation of element

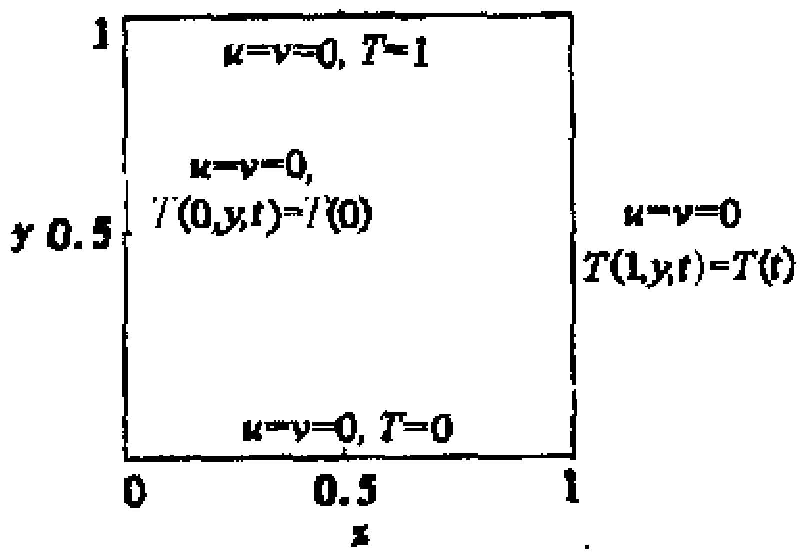

Fig.2 Schematic representation of physical model

Then, Problem (I_h^n) on K_i can be written as

$$\int_{K_i} u_{1h}^n v_h \mathrm{d}x\mathrm{d}y + k\mu\int_{K_i} \nabla u_{1h}^n \cdot \nabla v_h \mathrm{d}x\mathrm{d}y - k\int_{K_i} p_h^n \frac{\partial v_h}{\partial x}\mathrm{d}x\mathrm{d}y = \int_{K} u_{1h}^{n-1} v_h \mathrm{d}x\mathrm{d}y - k\int_{K_i} u_{1h}^{n-1}\frac{\partial u_{1h}^{n-1}}{\partial x} v_h \mathrm{d}x\mathrm{d}y - k\int_{K_i} u_{2h}^{n-1}\frac{\partial u_{1h}^{n-1}}{\partial y} v_h \mathrm{d}x\mathrm{d}y \qquad (\forall v_h \in P_1(K_i)), \tag{8}$$

$$\int_{K_i} u_{2h}^n v_h \mathrm{d}x\mathrm{d}y + \mu k\int_{K_i} \nabla u_{2h}^n \cdot \nabla v_h \mathrm{d}x\mathrm{d}y - k\int_{K_i} p_h^n \frac{\partial v_h}{\partial y}\mathrm{d}x\mathrm{d}y = \int_{K_i} u_{2h}^{n-1} v_h \mathrm{d}x\mathrm{d}y + k\lambda\int_{K_i} T_h^n v_h \mathrm{d}x\mathrm{d}y - k\int_{K_i} u_{1h}^{n-1}\frac{\partial u_{2h}^{n-1}}{\partial x} v_h \mathrm{d}x\mathrm{d}y - k\int_{K_i} u_{2h}^{n-1}\frac{\partial u_{2h}^{n-1}}{\partial y} v_h \mathrm{d}x\mathrm{d}y \qquad (\forall v_h \in P_1(K_i)), \tag{9}$$

$$\int_{K_i} \varphi_h \frac{\partial u_{1h}}{\partial x}\mathrm{d}x\mathrm{d}y + \int_{K_i} \varphi_h \frac{\partial u_{2h}}{\partial y}\mathrm{d}x\mathrm{d}y = 0 \qquad (\forall \varphi_h \in P_0(K_i)), \tag{10}$$

$$\int_{K} T_h^n \psi_h \mathrm{d}x\mathrm{d}y + k\int_{K_i} \nabla T_h^n \nabla \psi_h \mathrm{d}x\mathrm{d}y = \int_{K} T_h^{n-1}\psi_h \mathrm{d}x\mathrm{d}y - k\lambda\int_{K_i} u_{1h}^{n-1}\frac{\partial T_h^{n-1}}{\partial x}\psi_h \mathrm{d}x\mathrm{d}y - k\lambda\int_{K_i} u_{2h}^{n-1}\frac{\partial T_h^{n-1}}{\partial y}\psi_h \mathrm{d}x\mathrm{d}y \qquad (\forall \psi \in P_1(K_i)). \tag{11}$$

Note that (8) ~ (11) are a sufficient condition which Problem (I_h^n) holds. u_{jh}^n, p_h^n and T_h^n on K_i can denote by

$$u_{jh}^n = u_{ij1}^n\lambda_1 + u_{ij2}^n\lambda_2 + u_{ij3}^n\lambda_3 \qquad (j = 1,2), \tag{12}$$

$$T_h^n = T_{i1}^n\lambda_1 + T_{i2}^n\lambda_2 + T_{i3}^n\lambda_3, \tag{13}$$

$$p_h^n = \frac{1}{\Delta_{K_i}}\int_{K_i} p_i^n \mathrm{d}x\mathrm{d}y, \tag{14}$$

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where λ_1, λ_2 and λ_3 are barycentric coordinates (see Ref.[8] or Ref.[9]), $\Delta_{K_i} = h^2/2$ is the measure of triangle K_i. Then, by the properties of barycentric coordinate (see Ref.[8] or Ref.[9]), one can get

$$\frac{\partial}{\partial x} = \frac{-2}{h}\frac{\partial}{\partial\lambda_2}, \qquad \frac{\partial}{\partial y} = \frac{2}{h}\left(\frac{\partial}{\partial\lambda_2} - \frac{\partial}{\partial\lambda_1}\right). \tag{15}$$

Note that $\lambda_1 + \lambda_2 + \lambda_3 = 1$ on K_i, we have

$$\begin{cases} \dfrac{\partial\lambda_1}{\partial x} = 0, & \dfrac{\partial\lambda_2}{\partial x} = -\dfrac{2}{h}, \quad \dfrac{\partial\lambda_3}{\partial x} = \dfrac{2}{h}, \\ \dfrac{\partial\lambda_1}{\partial y} = -\dfrac{2}{h}, & \dfrac{\partial\lambda_2}{\partial y} = \dfrac{2}{h}, \quad \dfrac{\partial\lambda_3}{\partial x} = 0, \end{cases} \tag{16}$$

$$\frac{\partial T_h^n}{\partial x} = \frac{\partial T_h^n}{\partial\lambda_1}\frac{\partial\lambda_1}{\partial x} + \frac{\partial T_h^n}{\partial\lambda_2}\frac{\partial\lambda_2}{\partial x} = \frac{2}{h}(T_{i3}^n - T_{i2}^n), \tag{17}$$

$$\frac{\partial T_h^n}{\partial y} = \frac{\partial T_h^n}{\partial\lambda_1}\frac{\partial\lambda_1}{\partial y} + \frac{\partial T_h^n}{\partial\lambda_2}\frac{\partial\lambda_2}{\partial y} = \frac{2}{h}(T_{i2}^n - T_{i1}^n), \tag{18}$$

$$\nabla T_h^n = \frac{2}{h}(T_{i3}^n - T_{i2}^n, T_{i2}^n - T_{i1}^n). \tag{19}$$

Thus, (11) can be written as

$$\begin{aligned} &\int_{K_i}(T_{i1}^n\lambda_1 + T_{i2}^n\lambda_2 + T_{i3}^n\lambda_3)\lambda_m\mathrm{d}x\mathrm{d}y + \\ &\qquad \frac{2k}{h}\int_{K_i}\left[(T_{i3}^n - T_{i2}^n)\frac{\partial\lambda_m}{\partial x} + (T_{i2}^n - T_{i1}^n)\frac{\partial\lambda_m}{\partial y}\right]\mathrm{d}x\mathrm{d}y = \\ &\qquad \int_{K_i}(T_{i1}^{n-1}\lambda_1 + T_{i2}^{n-1}\lambda_2 + T_{i3}^{n-1}\lambda_3)\lambda_m\mathrm{d}x\mathrm{d}y - \\ &\qquad k\lambda\int_{K_i}(u_{i11}^{n-1}\lambda_1 + u_{i12}^{n-1}\lambda_2 + u_{i13}^{n-1}\lambda_3)\frac{2}{h}(T_{i3}^{n-1} - T_{i2}^{n-1})\lambda_m\mathrm{d}x\mathrm{d}y - \\ &\qquad k\lambda\int_{K_i}(u_{i21}^{n-1}\lambda_1 + u_{i22}^{n-1}\lambda_2 + u_{i23}^{n-1}\lambda_3)\frac{2}{h}(T_{i2}^{n-1} - T_{i1}^{n-1})\lambda_m\mathrm{d}x\mathrm{d}y \\ &\qquad\qquad (m = 1,2,3). \end{aligned} \tag{20}$$

Using the properties of barycentric coordinate again, one can get

$$\begin{pmatrix} 26 & -23 & 1 \\ -23 & 50 & -23 \\ 1 & -23 & 26 \end{pmatrix}\begin{pmatrix} T_{i1}^n \\ T_{i2}^n \\ T_{i3}^n \end{pmatrix} = \begin{pmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \end{pmatrix}, \tag{21}$$

where

$$\begin{aligned} F_{i1} = {} & 2T_{i1}^{n-1} + T_{i2}^{n-1} + T_{i3}^{n-1} + 2\lambda h(2u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(2u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1}), \end{aligned} \tag{22}$$

$$\begin{aligned} F_{i2} = {} & T_{i1}^{n-1} + 2T_{i2}^{n-1} + T_{i3}^{n-1} + 2\lambda h(u_{i21}^{n-1} + 2u_{i22}^{n-1} + u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(u_{i11}^{n-1} + 2u_{i12}^{n-1} + u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1}), \end{aligned} \tag{23}$$

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$$F_{i3} = T_{i1}^{n-1} + T_{i2}^{n-1} + 2T_{i3}^{n-1} + 2\lambda h(u_{i21}^{n-1} + u_{i22}^{n-1} + 2u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + 2\lambda h(u_{i11}^{n-1} + u_{i12}^{n-1} + 2u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1}). \tag{24}$$

Solving (21) can yield

$$\begin{aligned} T_{i1}^{n} = {} & [T_{i1}^{n-1} + T_{i2}^{n-1} + T_{i3}^{n-1} + 2\lambda h(u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1})]/4 + \\ & [771T_{i1}^{n-1} + 575T_{i2}^{n-1} + 479T_{i3}^{n-1} + \\ & 2\lambda h(771u_{i21}^{n-1} + 575u_{i22}^{n-1} + 479u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(771u_{i11}^{n-1} + 575u_{i12}^{n-1} + 479u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1})]/7300, \end{aligned} \tag{25}$$

$$\begin{aligned} T_{i2}^{n} = {} & [T_{i1}^{n-1} + T_{i2}^{n-1} + T_{i3}^{n-1} + 2\lambda h(u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1})]/4 + \\ & [575T_{i1}^{n-1} + 675T_{i2}^{n-1} + 575T_{i3}^{n-1} + \\ & 2\lambda h(575u_{i21}^{n-1} + 675u_{i22}^{n-1} + 575u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(575u_{i11}^{n-1} + 675u_{i12}^{n-1} + 575u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1})]/7300, \end{aligned} \tag{26}$$

$$\begin{aligned} T_{i3}^{n} = {} & [T_{i1}^{n-1} + T_{i2}^{n-1} + T_{i3}^{n-1} + 2\lambda h(u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1})]/4 + \\ & [479T_{i1}^{n-1} + 575T_{i2}^{n-1} + 771T_{i3}^{n-1} + \\ & 2\lambda h(479u_{i11}^{n-1} + 575u_{i12}^{n-1} + 771u_{i13}^{n-1})(T_{i1}^{n-1} - T_{i2}^{n-1}) + \\ & 2\lambda h(479u_{i21}^{n-1} + 575u_{i22}^{n-1} + 771u_{i23}^{n-1})(T_{i2}^{n-1} - T_{i3}^{n-1})]/7300. \end{aligned} \tag{27}$$

Using the same technique as (21), from (8) ~ (10), can yield

$$\begin{pmatrix} 2+24\mu & 1-24\mu & 1 & 0 & 0 & 0 & 0 \\ 1-24\mu & 2+48\mu & 1-24\mu & 0 & 0 & 0 & 12h \\ 1 & 1-24\mu & 2+24\mu & 0 & 0 & 0 & -12h \\ 0 & 0 & 0 & 2+24\mu & 1-24\mu & 1 & 12h \\ 0 & 0 & 0 & 1-24\mu & 2+48\mu & 1-24\mu & -12h \\ 0 & 0 & 0 & 1 & 1-24\mu & 2+24\mu & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} u_{i11}^{n} \\ u_{i12}^{n} \\ u_{i13}^{n} \\ u_{i21}^{n} \\ u_{i22}^{n} \\ u_{i23}^{n} \\ p_i^{n} \end{pmatrix} = \begin{pmatrix} f_{i11} \\ f_{i12} \\ f_{i13} \\ f_{i21} \\ f_{i22} \\ f_{i23} \\ 0 \end{pmatrix}, \tag{28}$$

where

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$$\begin{cases} f_{i11} = 2h(2u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1})(u_{i11}^{n-1} - u_{i12}^{n-1}) + 2h(2u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1}) \cdot \\ \qquad (u_{i12}^{n-1} - u_{i13}^{n-1}) + (2u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1}), \\ f_{i12} = 2h(u_{i21}^{n-1} + 2u_{i22}^{n-1} + u_{i23}^{n-1})(u_{i11}^{n-1} - u_{i12}^{n-1}) + 2h(u_{i11}^{n-1} + 2u_{i12}^{n-1} + u_{i13}^{n-1}) \cdot \\ \qquad (u_{i12}^{n-1} - u_{i13}^{n-1}) + (u_{i11}^{n-1} + 2u_{i12}^{n-1} + u_{i13}^{n-1}), \\ f_{i13} = 2h(u_{i21}^{n-1} + u_{i22}^{n-1} + 2u_{i23}^{n-1})(u_{i11}^{n-1} - u_{i12}^{n-1}) + 2h(u_{i11}^{n-1} + u_{i12}^{n-1} + 2u_{i13}^{n-1}) \cdot \\ \qquad (u_{i12}^{n-1} - u_{i13}^{n-1}) + (u_{i11}^{n-1} + u_{i12}^{n-1} + 2u_{i13}^{n-1}), \\ f_{i21} = \lambda(2T_{i1}^{n} + T_{i2}^{n} + T_{i3}^{n}) + 2h(2u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1})(u_{i21}^{n-1} - u_{i22}^{n-1}) + \\ \qquad 2h(2u_{i11}^{n-1} + u_{i12}^{n-1} + u_{i13}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1}) + (2u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1}), \\ f_{i22} = \lambda(T_{i1}^{n} + 2T_{i2}^{n} + T_{i3}^{n}) + 2h(u_{i21}^{n-1} + 2u_{i22}^{n-1} + u_{i23}^{n-1})(u_{i21}^{n-1} - u_{i22}^{n-1}) + \\ \qquad 2h(u_{i11}^{n-1} + 2u_{i12}^{n-1} + u_{i13}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1}) + (u_{i21}^{n-1} + 2u_{i22}^{n-1} + u_{i23}^{n-1}), \\ f_{i23} = \lambda(T_{i1}^{n} + T_{i2}^{n} + 2T_{i3}^{n}) + 2h(u_{i21}^{n-1} + u_{i22}^{n-1} + 2u_{i23}^{n-1})(u_{i21}^{n-1} - u_{i22}^{n-1}) + \\ \qquad 2h(u_{i11}^{n-1} + u_{i12}^{n-1} + 2u_{i13}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1}) + (u_{i21}^{n-1} + u_{i22}^{n-1} + 2u_{i23}^{n-1}). \end{cases}$$

Solving (28) can yield

$$\begin{aligned} p_i^n = {} & \frac{1}{18(1+36\mu)}[2(u_{i11}^{n-1} - u_{i12}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1}) + (u_{i12}^{n-1} - u_{i13}^{n-1})^2 + \\ & (u_{i21}^{n-1} - u_{i22}^{n-1})^2 + \mu(u_{i21}^{n-1} - 24u_{i22}^{n-1} + 23u_{i23}^{n-1})(u_{i12}^{n-1} - u_{i11}^{n-1}) + \\ & \mu(u_{i11}^{n-1} - 24u_{i12}^{n-1} + 23u_{i13}^{n-1})(u_{i13}^{n-1} - u_{i12}^{n-1}) + \\ & \mu(73u_{i21}^{n-1} - 24u_{i22}^{n-1} - 49u_{i23}^{n-1})(u_{i21}^{n-1} - u_{i22}^{n-1}) + \\ & \mu(73u_{i11}^{n-1} - 24u_{i12}^{n-1} - 49u_{i13}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1})] + \\ & \frac{1}{36h(1+36\mu)}[(u_{i12}^{n-1} - u_{i13}^{n-1}) + (u_{i21}^{n-1} - u_{i22}^{n-1}) + \lambda(T_{i1}^{n} - T_{i2}^{n}) - \\ & \mu(u_{i11}^{n-1} - 24u_{i12}^{n-1} + 23u_{i13}^{n-1}) + \mu\lambda(73T_{i1}^{n} - 24T_{i2}^{n} - 49T_{i3}^{n}) + \\ & \mu(73u_{i21}^{n-1} - 24u_{i22}^{n-1} - 49u_{i23}^{n-1})], \end{aligned} \tag{29}$$

$$\begin{aligned} u_{i13}^n = {} & \frac{2h(1+48\mu)}{3(1+24\mu)(1+36\mu)(1+72\mu)}[2(u_{i11}^{n-1} - u_{i12}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1}) + \\ & (u_{i21}^{n-1} - u_{i22}^{n-1})^2 + \mu(u_{i21}^{n-1} - 24u_{i22}^{n-1} + 23u_{i23}^{n-1})(u_{i12}^{n-1} - u_{i11}^{n-1}) + \\ & (u_{i12}^{n-1} - u_{i13}^{n-1})^2 + \mu(u_{i11}^{n-1} - 24u_{i12}^{n-1} + 23u_{i13}^{n-1})(u_{i13}^{n-1} - u_{i12}^{n-1}) + \\ & \mu(73u_{i21}^{n-1} - 24u_{i22}^{n-1} - 49u_{i23}^{n-1})(u_{i21}^{n-1} - u_{i22}^{n-1}) + \\ & \mu(73u_{i11}^{n-1} - 24u_{i12}^{n-1} - 49u_{i13}^{n-1})(u_{i22}^{n-1} - u_{i23}^{n-1})] + \\ & \frac{1+48\mu}{3(1+24\mu)(1+36\mu)(1+72\mu)}[(u_{i12}^{n-1} - u_{i13}^{n-1}) + (u_{i21}^{n-1} - u_{i22}^{n-1}) + \\ & \lambda(T_{i1}^{n} - T_{i2}^{n}) - \mu(u_{i11}^{n-1} - 24u_{i12}^{n-1} + 23u_{i13}^{n-1}) + \\ & \mu\lambda(73T_{i1}^{n} - 24T_{i2}^{n} - 49T_{i3}^{n}) + \mu(73u_{i21}^{n-1} - 24u_{i22}^{n-1} - 49u_{i23}^{n-1})] + \\ & \frac{1}{1+72\mu}[2hu_{i23}^{n-1}(u_{i11}^{n-1} - u_{i12}^{n-1}) + 2hu_{i13}^{n-1}(u_{i12}^{n-1} - u_{i13}^{n-1}) + u_{i13}^{n-1}] + \\ & \frac{24\mu}{1+72\mu}[2h(u_{i21}^{n-1} + u_{i22}^{n-1} + u_{i23}^{n-1}) + (u_{i11}^{n-1} - u_{i12}^{n-1}) + \end{aligned}$$

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$$\begin{aligned}
&2h(u^{n-1}_{i11} + u^{n-1}_{i12} + u^{n-1}_{i13})(u^{n-1}_{i12} - u^{n-1}_{i13}) + (u^{n-1}_{i11} + u^{n-1}_{i12} + u^{n-1}_{i13})] + \\
&\frac{24\mu}{(1+24\mu)(1+72\mu)}[2h(u^{n-1}_{i23} - u^{n-1}_{i21})(u^{n-1}_{i11} - u^{n-1}_{i12}) + \\
&2h(u^{n-1}_{i13} - u^{n-1}_{i11})(u^{n-1}_{i12} - u^{n-1}_{i13}) + u^{n-1}_{i13} - u^{n-1}_{i11}],
\end{aligned} \tag{30}$$

$$\begin{aligned}
u^n_{i12} = {} & -2u^n_{i13} + \frac{2h}{3(1+24\mu)(1+36\mu)}[2(u^{n-1}_{i11} - u^{n-1}_{i12})(u^{n-1}_{i22} - u^{n-1}_{i23}) + \\
&(u^{n-1}_{i21} - u^{n-1}_{i22})^2 + \mu(u^{n-1}_{i21} - 24u^{n-1}_{i22} + 23u^{n-1}_{i23})(u^{n-1}_{i12} - u^{n-1}_{i11}) + \\
&(u^{n-1}_{i12} - u^{n-1}_{i13})^2 + \mu(u^{n-1}_{i11} - 24u^{n-1}_{i12} + 23u^{n-1}_{i13})(u^{n-1}_{i13} - u^{n-1}_{i12}) + \\
&\mu(73u^{n-1}_{i21} - 24u^{n-1}_{i22} - 49u^{n-1}_{i23})(u^{n-1}_{i21} - u^{n-1}_{i22}) + \\
&\mu(73u^{n-1}_{i11} - 24u^{n-1}_{i12} - 49u^{n-1}_{i13})(u^{n-1}_{i22} - u^{n-1}_{i23})] + \\
&\frac{1}{3(1+24\mu)(1+36\mu)}[(u^{n-1}_{i12} - u^{n-1}_{i13}) + (u^{n-1}_{i21} - u^{n-1}_{i22}) + \\
&\lambda(T^n_{i1} - T^n_{i2}) - \mu(u^{n-1}_{i11} - 24u^{n-1}_{i12} + 23u^{n-1}_{i13}) + \\
&\mu\lambda(73T^n_{i1} - 24T^n_{i2} - 49T^n_{i3}) + \mu(73u^{n-1}_{i21} - 24u^{n-1}_{i22} - 49u^{n-1}_{i23})] + \\
&2h(u^{n-1}_{i12} + u^{n-1}_{i12} + u^{n-1}_{i13})(u^{n-1}_{i11} - u^{n-1}_{i12}) + u^{n-1}_{i11} + u^{n-1}_{i12} + u^{n-1}_{i13} + \\
&2h(u^{n-1}_{i11} + u^{n-1}_{i12} + u^{n-1}_{i13})(u^{n-1}_{i12} - u^{n-1}_{i13}) + \\
&\frac{1}{1+24\mu}[2h(u^{n-1}_{i23} - u^{n-1}_{i21})(u^{n-1}_{i11} - u^{n-1}_{i12}) + \\
&2h(u^{n-1}_{i13} - u^{n-1}_{i11})(u^{n-1}_{i12} - u^{n-1}_{i13}) + u^{n-1}_{i13} - u^{n-1}_{i11}],
\end{aligned} \tag{31}$$

$$\begin{aligned}
u^n_{i11} = {} & u^n_{i13} - \frac{2h}{3(1+24\mu)(1+36\mu)}[2(u^{n-1}_{i11} - u^{n-1}_{i12})(u^{n-1}_{i22} - u^{n-1}_{i23}) + \\
&(u^{n-1}_{i21} - u^{n-1}_{i22})^2 + \mu(u^{n-1}_{i21} - 24u^{n-1}_{i22} + 23u^{n-1}_{i23})(u^{n-1}_{i12} - u^{n-1}_{i11}) + \\
&(u^{n-1}_{i12} - u^{n-1}_{i13})^2 + \mu(u^{n-1}_{i11} - 24u^{n-1}_{i12} + 23u^{n-1}_{i13})(u^{n-1}_{i13} - u^{n-1}_{i12}) + \\
&\mu(73u^{n-1}_{i21} - 24u^{n-1}_{i22} - 49u^{n-1}_{i23})(u^{n-1}_{i21} - u^{n-1}_{i22}) + \\
&\mu(73u^{n-1}_{i11} - 24u^{n-1}_{i12} - 49u^{n-1}_{i13})(u^{n-1}_{i22} - u^{n-1}_{i23})] - \\
&\frac{1}{3(1+24\mu)(1+36\mu)}[(u^{n-1}_{i12} - u^{n-1}_{i13}) + (u^{n-1}_{i21} - u^{n-1}_{i22}) + \lambda(T^n_{i1} - T^n_{i2}) - \\
&\mu(u^{n-1}_{i11} - 24u^{n-1}_{i12} + 23u^{n-1}_{i13}) + \mu\lambda(73T^n_{i1} - 24T^n_{i2} - 49T^n_{i3}) + \\
&\mu(73u^{n-1}_{i21} - 24u^{n-1}_{i22} - 49u^{n-1}_{i23})] - \\
&\frac{1}{1+24\mu}[2h(u^{n-1}_{i23} - u^{n-1}_{i21})(u^{n-1}_{i11} - u^{n-1}_{i12}) + \\
&2h(u^{n-1}_{i13} - u^{n-1}_{i11})(u^{n-1}_{i12} - u^{n-1}_{i13}) + u^{n-1}_{i13} - u^{n-1}_{i11}],
\end{aligned} \tag{32}$$

$$\begin{aligned}
u^n_{i23} = {} & \frac{16h\mu}{(1+24\mu)(1+36\mu)(1+72\mu)}[2(u^{n-1}_{i11} - u^{n-1}_{i12})(u^{n-1}_{i22} - u^{n-1}_{i23}) + \\
&(u^{n-1}_{i21} - u^{n-1}_{i22})^2 + \mu(u^{n-1}_{i21} - 24u^{n-1}_{i22} + 23u^{n-1}_{i23})(u^{n-1}_{i12} - u^{n-1}_{i11}) + \\
&(u^{n-1}_{i12} - u^{n-1}_{i13})^2 + \mu(u^{n-1}_{i11} - 24u^{n-1}_{i12} + 23u^{n-1}_{i13})(u^{n-1}_{i13} - u^{n-1}_{i12}) + \\
&\mu(73u^{n-1}_{i21} - 24u^{n-1}_{i22} - 49u^{n-1}_{i23})(u^{n-1}_{i21} - u^{n-1}_{i22}) + \\
&\mu(73u^{n-1}_{i11} - 24u^{n-1}_{i12} - 49u^{n-1}_{i13})(u^{n-1}_{i22} - u^{n-1}_{i23})] + \\
&\frac{8\mu}{(1+24\mu)(1+36\mu)(1+72\mu)}[(u^{n-1}_{i12} - u^{n-1}_{i13}) + (u^{n-1}_{i21} - u^{n-1}_{i22}) + \\
&\lambda(T^n_{i1} - T^n_{i2}) - \mu(u^{n-1}_{i11} - 24u^{n-1}_{i12} + 23u^{n-1}_{i13}) +
\end{aligned}$$

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$$
\begin{aligned}
&\mu\lambda(73T_{i1}^{n}-24T_{i2}^{n}-49T_{i3}^{n})+\mu(73u_{i21}^{n-1}-24u_{i22}^{n-1}-49u_{i23}^{n-1})]+\\
&\frac{1}{1+72\mu}[\lambda T_{i3}^{n}+u_{i23}^{n-1}+2hu_{i23}^{n-1}(u_{i21}^{n-1}-u_{i22}^{n-1})+2hu_{i13}^{n-1}(u_{i22}^{n-1}-u_{i23}^{n-1})]+\\
&\frac{24\mu}{1+72\mu}[\lambda(T_{i1}^{n}+T_{i2}^{n}+T_{i3}^{n})+2h(u_{i21}^{n-1}+u_{i22}^{n-1}+u_{i23}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&2h(u_{i11}^{n-1}+u_{i12}^{n-1}+u_{i13}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+u_{i21}^{n-1}+u_{i22}^{n-1}+u_{i23}^{n-1}]+\\
&\frac{24\mu}{(1+24\mu)(1+72\mu)}[\lambda(T_{i3}^{n}-T_{i1}^{n})+2h(u_{i23}^{n-1}-u_{i21}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&2h(u_{i13}^{n-1}-u_{i11}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+u_{i23}^{n-1}-u_{i21}^{n-1}],
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
u_{i22}^{n}={}&-2u_{i23}^{n}+\frac{2h}{3(1+24\mu)(1+36\mu)}[2(u_{i11}^{n-1}-u_{i12}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+\\
&(u_{i21}^{n-1}-u_{i22}^{n-1})^{2}+\mu(u_{i21}^{n-1}-24u_{i22}^{n-1}+23u_{i23}^{n-1})(u_{i12}^{n-1}-u_{i11}^{n-1})+\\
&(u_{i12}^{n-1}-u_{i13}^{n-1})^{2}+\mu(u_{i11}^{n-1}-24u_{i12}^{n-1}+23u_{i13}^{n-1})(u_{i13}^{n-1}-u_{i12}^{n-1})+\\
&\mu(73u_{i21}^{n-1}-24u_{i22}^{n-1}-49u_{i23}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&\mu(73u_{i11}^{n-1}-24u_{i12}^{n-1}-49u_{i13}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})]+\\
&\frac{1}{3(1+24\mu)(1+36\mu)}[(u_{i12}^{n-1}-u_{i13}^{n-1})+(u_{i21}^{n-1}-u_{i22}^{n-1})+\lambda(T_{i1}^{n}-T_{i2}^{n})-\\
&\mu(u_{i11}^{n-1}-24u_{i12}^{n-1}+23u_{i13}^{n-1})+\mu\lambda(73T_{i1}^{n}-24T_{i2}^{n}-49T_{i3}^{n})+\\
&\mu(73u_{i21}^{n-1}-24u_{i22}^{n-1}-49u_{i23}^{n-1})]+2h(u_{i21}^{n-1}+u_{i22}^{n-1}+u_{i23}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&2h(u_{i11}^{n-1}+u_{i12}^{n-1}+u_{i13}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+u_{i21}^{n-1}+u_{i22}^{n-1}+u_{i23}^{n-1}+\\
&\lambda(T_{i1}^{n}+T_{i2}^{n}+T_{i3}^{n})+\frac{1}{1+24\mu}[2h(u_{i23}^{n-1}-u_{i21}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&\lambda(T_{i3}^{n}-T_{i1}^{n})+2h(u_{i13}^{n-1}-u_{i11}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+u_{i23}^{n-1}-u_{i21}^{n-1}],
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
u_{i21}^{n}={}&u_{i23}^{n}-\frac{2h}{3(1+24\mu)(1+36\mu)}[2(u_{i11}^{n-1}-u_{i12}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+\\
&(u_{i21}^{n-1}-u_{i22}^{n-1})^{2}+\mu(u_{i21}^{n-1}-24u_{i22}^{n-1}+23u_{i23}^{n-1})(u_{i12}^{n-1}-u_{i11}^{n-1})+\\
&(u_{i12}^{n-1}-u_{i13}^{n-1})^{2}+\mu(u_{i11}^{n-1}-24u_{i12}^{n-1}+23u_{i13}^{n-1})(u_{i13}^{n-1}-u_{i12}^{n-1})+\\
&\mu(73u_{i21}^{n-1}-24u_{i22}^{n-1}-49u_{i23}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&\mu(73u_{i11}^{n-1}-24u_{i12}^{n-1}-49u_{i13}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})]-\\
&\frac{1}{3(1+24\mu)(1+36\mu)}[(u_{i12}^{n-1}-u_{i13}^{n-1})+(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&\lambda(T_{i1}^{n}-T_{i2}^{n})-\mu(u_{i11}^{n-1}-24u_{i12}^{n-1}+23u_{i13}^{n-1})+\\
&\mu\lambda(73T_{i1}^{n}-24T_{i2}^{n}-49T_{i3}^{n})+\mu(73u_{i21}^{n-1}-24u_{i22}^{n-1}-49u_{i23}^{n-1})]-\\
&\frac{1}{12+24\mu}[\lambda(T_{i3}^{n}-T_{i1}^{n})+2h(u_{i23}^{n-1}-u_{i21}^{n-1})(u_{i21}^{n-1}-u_{i22}^{n-1})+\\
&2h(u_{i13}^{n-1}-u_{i11}^{n-1})(u_{i22}^{n-1}-u_{i23}^{n-1})+u_{i23}^{n-1}-u_{i21}^{n-1}].
\end{aligned}
\tag{35}
$$

Equations (25) ~ (27) and (29) ~ (35) make together up a lowest order finite difference scheme based on MFE method for non-stationary natural convection Problem (Ⅰ). Inserting them into Eqs.(12) ~ (14) can yield the formulae of $u^{h}=(u_{1h}^{n},u_{2h}^{n})$, p_{h}^{n}, and T_{h}^{n} on K_{i}.

3 Numerical Example

In this section, we present two examples to simulate the close square cavity showing the

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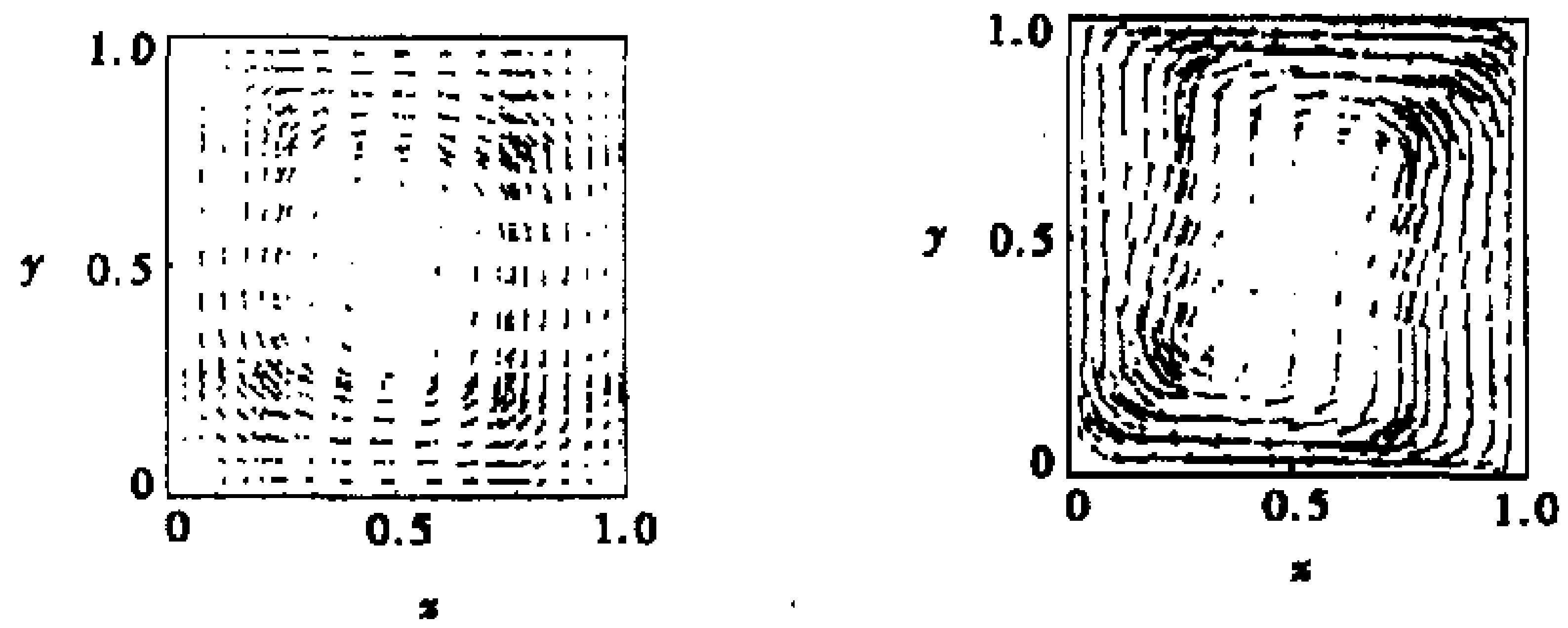
efficient of our lowest order difference scheme based on MFE method. The above side and below side of the close square cavity are two walls of constant temperature, and the left side and right side are two insulated walls, see Fig.2. And there is only air in the close square cavity. Since there is difference of temperature on the above side and below side in the close square cavity, there is natural convection in the close square cavity. Let us take the domain of the close square cavity as $\bar{\Omega} = [0,1] \times [0,1]$, and cut $\bar{\Omega}$ into 100×100 small squares, and then link the left above vertex and right below vertesx of each square to form two triangles. Thus $h = k = 0.01$. According to the array of total knots, the values of numerical solution (u_{1h}^n, T_h^n) at all knots can be written as

$$\begin{cases} u_{1h}^n(0,l) = u_{1h}^n(100,l) = u_{2h}^n(0,l) = u_{2h}^n(100,l) = 0 \quad (0 \leqslant l \leqslant 100); \\ u_{1h}^n(s,0) = u_{1h}^n(s,100) = u_{2h}^n(s,0) = u_{2h}^n(s,100) = 0 \quad (0 \leqslant s \leqslant 100); \\ u_{jh}^n(s,l) = [u_{2l+100(s-1),j,2}^n + u_{2l+100(s-1)+1,j,3}^n + u_{2l+100(s-1)+2,j,1}^n + \\ \qquad u_{2l+100s-1,j,1}^n + u_{2l+100s,j,3}^n + u_{2l+100s+1,j,2}^n]/6 \\ \qquad (1 \leqslant s \leqslant 99, 1 \leqslant l \leqslant 99, j = 1,2); \end{cases}$$

$$\begin{cases} T_h^n(0,l) = 0;\ T_h^n(100,l) = 0 \quad (0 \leqslant l \leqslant 100); \\ T_h^n(s,0) = 0;\ T_h^n(s,100) = 1 \quad (0 \leqslant s \leqslant 100); \\ T_h^n(s,l) = [T_{2l+100(s-1),2}^n + T_{2l+100(s-1)+1,3}^n + T_{2l+100(s-1)+2,1}^n + \\ \qquad T_{2l+100s-1,1}^n + T_{2l+100s}^n + T_{2l+100s+1,2}^n]/6 \\ \qquad (1 \leqslant s \leqslant 99, 1 \leqslant l \leqslant 99). \end{cases}$$

Using the above formulae we compute out the numerical solutions of velocity, temperature, and pressure together when $\mu = 0.000\,01$ (i.e., $Ra = 10\,000$) and $\mu = 0.000\,001$ (i.e., $Ra = 100\,000$), see Figs.3 ~ 5.



$Ra = 10\,000$ $\qquad$ $Ra = 100\,000$

Fig.3 Velocity vector at different Ra-value

Conclusion Our numerical results, which are only used for a first degree element for the velocity and the temperature, and a zero degree element for pressure, are very ideal in comparison with those of other finite difference scheme (see Refs.[1 ~ 2]). Especially, our finite difference scheme based mixed finite element method can deal with very big Ra-number cases, for example, $Ra = 100\,000$, and can find together the numerical solutions of the velocity, the pressure, and the temperature.

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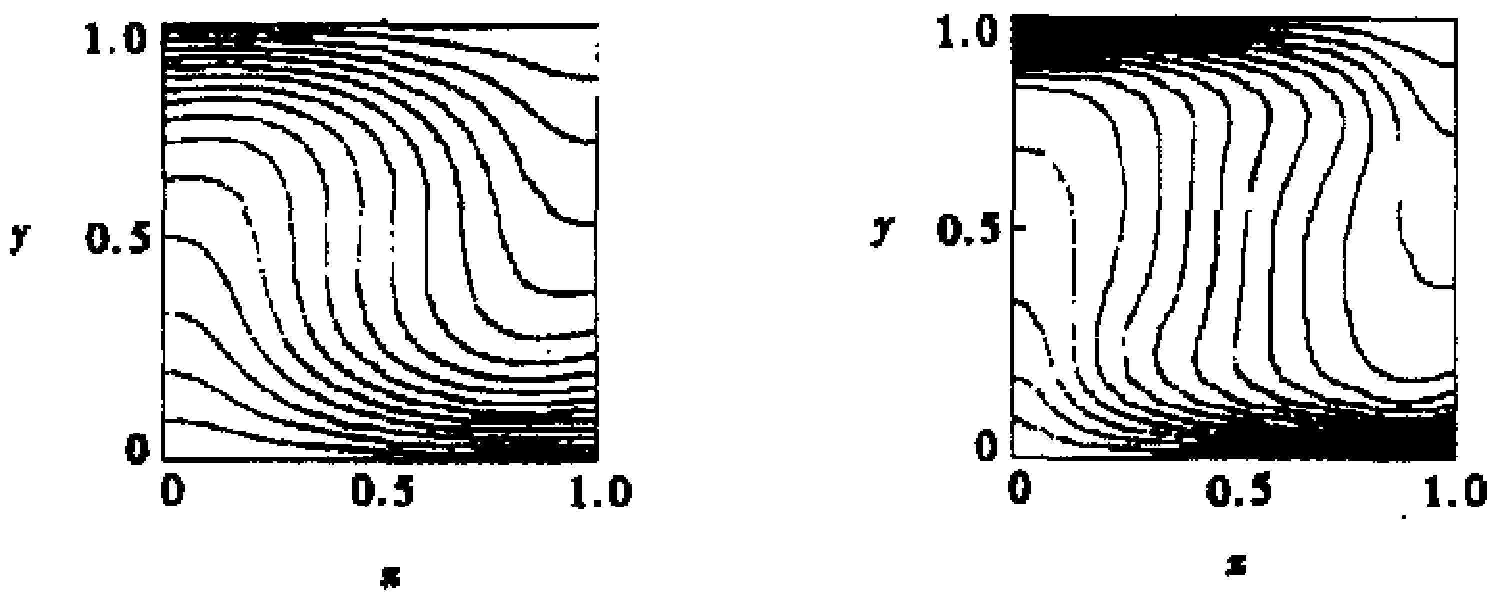
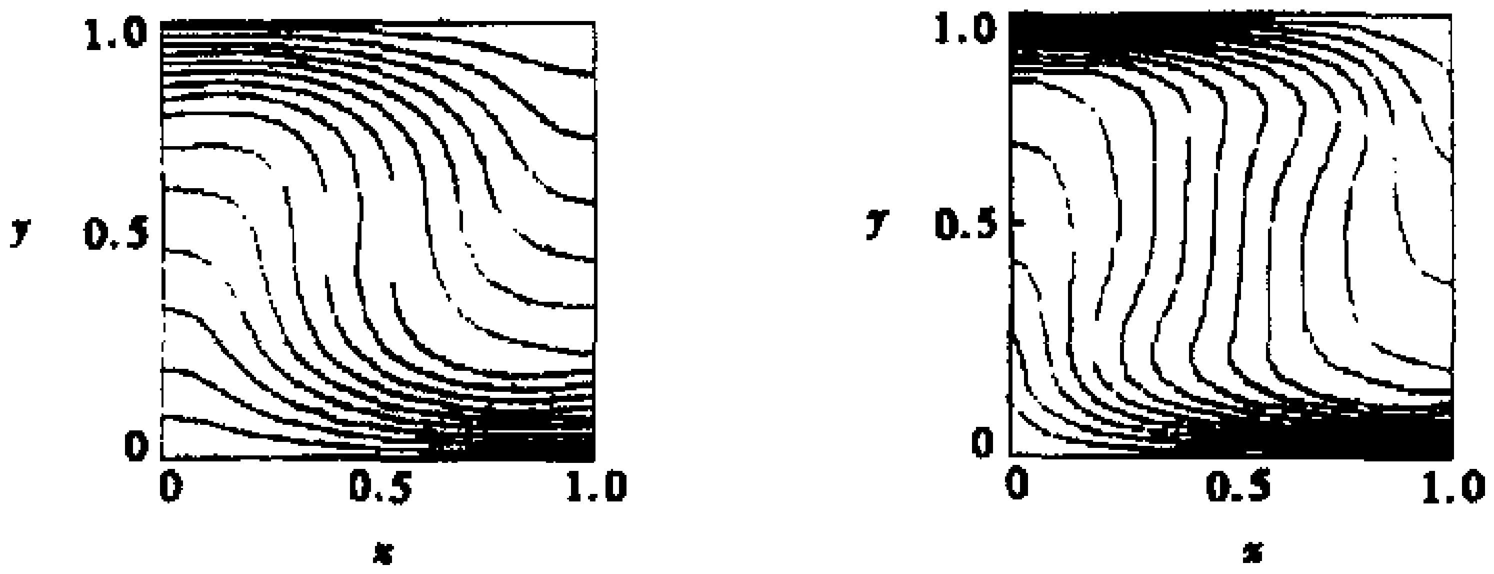


Ra = 10 000 Ra = 100 000

Fig.4 Temperature field at different Ra-value



Ra = 10 000 Ra = 100 000

Fig.5 Pressure field at different Ra-value

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万方数据